Griffiths.

8.1 (a), (b), (c) are easily seen, we do (d) below.

$$\langle |\mathcal{M}|^2 \rangle = \frac{Q^2 g_e^4}{(p_1+p_2)^4} 8\Big[(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) + (mc)^2(p_3\cdot p_4)$$
$$+ (Mc)^2(p_1\cdot p_2)$$
$$+ 2(mc)^2(Mc)^2\Big].$$

$$(p_1+p_2)^4 = (E)^4/c^4$$

$$(p_1\cdot p_3) = \frac{E^2}{4c^2} - \vec{p}_1\cdot\vec{p}_3, \quad (p_1\cdot p_4) = \frac{E^2}{4c^2} + \vec{p}_1\cdot\vec{p}_3$$

$$(p_2\cdot p_4) = \frac{E^2}{4c^2} - \vec{p}_1\cdot\vec{p}_3, \quad (p_2\cdot p_3) = \frac{E^2}{4c^2} + \vec{p}_1\cdot\vec{p}_3$$

$$(p_3\cdot p_4) = \frac{E^2}{4c^2} + |p_3|^2, \quad (p_1\cdot p_2) = \frac{E^2}{4c^2} + |p_1|^2.$$

$$\langle \mathcal{M}^2 \rangle = (Q^2 g_e^4)\left(\frac{8c^4}{E^4}\right)\Big[\left[\frac{E^2}{4c^2}\right]^2 + (\vec{p}_1\cdot\vec{p}_3)^2 + \left[\frac{E^2}{4c^2}\right]^2 + \left(\vec{p}_1\cdot\vec{p}_3\right)^2$$
$$+ (mc)^2\left[\frac{E^2}{4c^2} + |p_3|^2\right] + (Mc)^2\left[\frac{E^2}{4c^2} + |p_1|^2\right]$$
$$+ 2(mc)^2(Mc)^2\Big].$$

$$= (Q^2 g_e^4)\left[\frac{8c^4}{E^4}\right]\Big\{\frac{2E^4}{16c^4} + 2(\vec{p}_1\cdot\vec{p}_3)^2 + \frac{E^2m^2}{4} + m^2c^2|p_3|^2$$
$$+ \frac{E^2M^2}{4} + M^2c^2|p_1|^2 + 2m^2M^2c^4\Big].$$

$$= Q^2 g_e^4\Big[1 + \frac{16c^4(\vec{p}_1\cdot\vec{p}_3)^2}{E^4} + \frac{2m^2c^4}{E^2} + \frac{8m^2c^6|p_3|^2}{E^4}$$
$$+ \frac{2M^2c^4}{E^2} + \frac{8M^2c^6|p_1|^2}{E^4}$$
$$+ \frac{16m^2M^2c^8}{E^4}$$

$$= Q^2 q_e^4 \left[1 + \frac{2m^2c^4}{E^2} + \frac{2M^2c^4}{E^2} + \frac{1}{E^4}[\cdots]\right]$$

$$c^4(\vec{p}_1 \cdot \vec{p}_3)^2 = \cos^2\theta |p_1|^2 |p_3|^2 c^4, \quad |p_1|^2c^2 = \frac{E^2}{4} - m^2c^4, \quad |p_3|^2c^2 = \frac{E^2}{4} - M^2c^4.$$

$$[\cdots] = \left[16c^4(\vec{p}_1\cdot\vec{p}_3)^2 + 8m^2c^6|p_3|^2 + 8M^2c^6|p_1|^2 + 16m^2M^2c^8\right]$$

$$= \left[16\cos^2\theta\left[\frac{E^2}{4} - m^2c^4\right]\left[\frac{E^2}{4} - M^2c^4\right] + 8m^2c^4\left[\frac{E^2}{4} - M^2c^4\right]\right.$$

$$\left. + 8M^2c^4\left[\frac{E^2}{4} - m^2c^4\right] + 16m^2M^2c^8\right]$$

$$= \left[\cos^2\theta\left[E^2 - \frac{m^2c^4}{4}\right]\left[E^2 - \frac{M^2c^4}{4}\right] + 2m^2c^4E^2 + 2M^2c^4E^2\right.$$

$$\left. \cancel{- 8M^2m^2c^8} \cancel{- 8M^2m^2c^8} \cancel{+ 16m^2M^2c^8}\right]$$

$$\frac{1}{E^4}[\cdots] = \cos^2\theta\left[1 - \frac{m^2c^4}{4E^2}\right]\left[1 - \frac{M^2c^4}{4E^2}\right] + \frac{2m^2c^4}{E^2} + \frac{2M^2c^4}{E^2}$$

$$\Rightarrow \langle \mathcal{M}^2 \rangle = Q^2 q_e^4 \left\{1 + \frac{4m^2c^4}{E^2} + \frac{4M^2c^4}{E^2} + \cos^2\theta\left[1 - \frac{m^2c^4}{4E^2}\right]\left[1 - \frac{M^2c^4}{4E^2}\right]\right.$$

$$= Q^4 q_e^4 \left[1 + \cancel{\frac{2m^2c^4}{}} \left(\frac{2mc^2}{E}\right)^2 + \left(\frac{2Mc^2}{E}\right)^2 + \cos^2\theta\left[1 - \left(\frac{mc^2}{2E}\right)^2\right]\left[1 - \left(\frac{Mc^2}{2E}\right)^2\right]\right]$$

This is equivalent to (8.4) upto a redefinition of $E \to 2E$.